

# WDFS 24

Wave Dynamics and Fluid-Structure Interactions

Como, May 27-31, 2024

# Invited speakers

## Course lecturers

Massimiliano Berti (SISSA Trieste)

Francisco Gancedo (University of Seville)

## Senior speakers

Luigi Carlo Berselli (University of Pisa)

Luca Bisconti (University of Florence)

Denis Bonheure (Free University of Brussels)

Elia Bruè (Bocconi University Milan)

Tristan Buckmaster (New York University)

Livia Corsi (Roma Tre University)

Michele Coti Zelati (Imperial College London)

Giovanni P. Galdi (University of Pittsburgh)

Andrea Giorgini (Polytechnic University of Milan)

Zaher Hani (University of Michigan)

Matthieu Hillairet (University of Montpellier)

Taoufik Hmidi (New York University Abu Dhabi)

Alexander Kiselev (Duke University)

Alberto Maspero (SISSA Trieste)

Anna Laura Mazzucato (Penn State University)

Stefano Modena (Gran Sasso Science Institute)

Paolo Secchi (University of Brescia)

## Junior speakers

Edoardo Bocchi (Polytechnic University of Milan)

Matteo Caggio (Czech Academy of Sciences)

Luca Franzoi (University of Milan)

Ricardo Grande Izquierdo (SISSA Trieste)

Camilla Nobili (University of Surrey)

Xiao Ren (Peking University)

## Schedule of the Workshop

	Monday 27 <sup>th</sup>	Tuesday 28 <sup>th</sup>	Wednesday 29 <sup>th</sup>	Thursday 30 <sup>th</sup>	Friday 31 <sup>st</sup>
9:00-10:00	Berti	Gancedo	Berti	Gancedo	Berti
10:00-11:00	Gancedo	Berti	Gancedo	Berti	Gancedo
11:00-11:30	<i>Coffee break</i>	<i>Coffee break</i>	<i>Coffee break</i>	<i>Coffee break</i>	<i>Coffee break</i>
11:30-12:15	Buckmaster	Berselli	Bruè	Mazzucato	Coti Zelati
12:15-13:00	Hani	Hmidi	Hillairet	Maspero	Kiselev
13:00-15:00	<i>Lunch</i>	<i>Lunch</i>	<i>Light lunch</i>	<i>Lunch</i>	<i>Light lunch</i>
15:00-15:45	Secchi	Bonheure		Giorgini	
15:45-16:30	Modena	Corsi		Bisconti	
16:30-17:00	<i>Posters&amp;Coffee</i>	<i>Posters&amp;Coffee</i>		<i>Posters&amp;Coffee</i>	
17:00-17:30	Caggio	Franzoi		Nobili	
17:30-18:00	Bocchi	Grande Izquierdo		Ren	
19:30-21:30			<i>Social Dinner</i>		

The social dinner will take place at the Bellavista Boutique Hotel and Restaurant in Brunate

[www.bellavistabrunate.com](http://www.bellavistabrunate.com)

which can be reached by funicular railway from Como. We advise you to bring a light pullover with you, in case the dinner takes place outdoors.

# Courses

## Unstable Stokes Waves

*Massimiliano BERTI, SISSA Trieste*

A question of fundamental physical importance is to establish if Stokes waves, namely periodic traveling solutions of the pure gravity water waves equations, are stable or unstable. In the '60 Benjamin, Feir, Whitham Lighthill and Zakharov discovered, through experiments and formal arguments, that small amplitude periodic Stokes waves in sufficiently deep water are unstable when subject to long-wave perturbations and proposed an heuristic mechanism that leads to the disintegration of wave trains. In the '80 numerical work of McLean detected a different type of modulation instability far away from the origin of the complex plane. From a mathematical point of view this amounts to study the spectral bands that leave the imaginary axis of a Hamiltonian and reversible pseudo-differential matrix valued operator with periodic coefficients. In this course I will present recent mathematically rigorous results about the band spectrum of the water waves equations linearized at the Stokes solitary waves.

This material is based on joint works with A. Maspero, P. Ventura and L. Corsi.

# Dynamics of coherent structures for incompressible fluids

*Francisco GANCEDO, University of Seville*

In this course, we will discuss some recent results on the dynamics of coherent structures in incompressible fluids, focusing on interface dynamics and vortex filaments. Over the five sessions, we will cover the following topics:

- Lecture 1) We will explore gravity-driven flow for density patches governed by Stokes' law, obtaining global-in-time regularity, stability, and instability results.
- Lecture 2) We will deal with the full 2D Navier-Stokes equations, presenting new global-in-time regularity results for fluids with different viscosities and densities.
- Lecture 3) We will introduce new results for the Muskat problem, where the fluid is filtered in a porous medium. We will show the main ideas to achieve global-in-time well-posedness in the gravity-stable case for initial Lipschitz graphs of arbitrary size.
- Lecture 4) We will study the Muskat problem with surface tension, obtaining nonlinear stability for bubbles using a recent new approach. If time permits, we will discuss new results on Surface Quasi-Geostrophic fronts.
- Lecture 5) We will introduce the dynamics of vortex filaments for 3D Navier-Stokes and Euler flows. We will demonstrate global regularity for 3D Navier-Stokes helical vortex filaments and the existence of 3D Euler flows originating from circular vortex filaments.

# Senior talks

## Fourier-Galerkin approximation of the solutions of the 2D Euler equations with bounded vorticity

*Luigi Carlo BERSELLI, University of Pisa*

We study semi and fully discrete approximation of the 2D Euler equations for ideal homogeneous fluids. We focus on spectral methods and discuss rates of convergence of velocity and vorticity under different assumptions on the smoothness of the data.

## Extension criterion for a 3D viscous Tropical Climate Model with Damping

*Luca BISCONTI, University of Florence*

In this talk we consider the viscous Tropical Climate Model (TCM) in the whole three-dimensional space. This model is a coupled system of the barotropic and first baroclinic modes of the velocity and typical mid-tropospheric temperature. Taking into account a modified version of the viscous TCM, with nonlinear damping terms in the equations for the barotropic mode  $u$  and the first baroclinic mode  $v$  of the velocity, we establish an extension criterion for the global well-posedness of the corresponding strong solutions, in terms of suitable homogeneous Besov norms for  $u$  and  $v$ .

## Time-Periodic Body-Liquid Interaction Problem: forced oscillations and Hopf bifurcation scenario

*Denis BONHEURE, Free University of Brussels*

I will give an overview of several results obtained recently concerning the motion of a rigid body subject to an undamped elastic restoring force, in the stream of a viscous liquid under the action of a prescribed flow at large distances. The framework is a 3D external domain where a uniform flow is given at spatial infinity. When this imposed flow is steady, we investigate a possible dynamic instability through Hopf bifurcation. When it is periodic and unsteady, we discuss the existence of at least one periodic excited state.

## Non-Uniqueness in Two-Dimensional Euler Equations

*Elia BRUÈ, Bocconi University Milan*

In 1962, Yudovich established the well-posedness of the two-dimensional incompressible Euler equations within the class of solutions with bounded vorticity. Since then, a central unresolved problem has been the question of uniqueness within the broader class of solutions with  $L^p$ -vorticities. Recent years have witnessed significant progress in this investigation. In my talk, I aim to provide an overview of these developments and highlight recent results obtained thanks to the convex integration method.

**TBA**

*Tristan BUCKMASTER, New York University*

**TBA**

*Livia CORSI, Roma Tre University*

### **Stability thresholds in three dimensional stratified fluids**

*Michele COTI ZELATI, Imperial College London*

The stability of shear flows in the fluid mechanics is an old problem dating back to the famous Reynolds experiments in 1883. The question is to quantify the size of the basin of attraction of equilibria of the Navier-Stokes equations depending on the viscosity parameters, giving rise to the so-called stability threshold. In the case of a three-dimensional homogeneous fluid, it is known that the Couette flow has a stability threshold proportional to the viscosity, and this is sharp in view of a linear instability mechanism known as the lift-up effect. In this talk, I will explain how to exploit stratification (i.e. non-homogeneity in the fluid density) to improve this bound: the coupling between density and velocity gives rise to oscillations, which suppress the lift-effect. This can be captured at the linear level in an explicit manner, and at the nonlinear level by combining sharp energy estimates with suitable dispersive estimates.

### **Global regularity and asymptotic stabilization for the Navier-Stokes-Cahn-Hilliard model with unmatched densities**

*Andrea GIORGINI, Polytechnic University of Milan*

The Abels-Garcke-Grün (AGG) system is a thermodynamically consistent diffuse interface model for fluid mixtures. It describes the motion of two viscous and incompressible fluids with unmatched densities undergoing phase separation. The AGG model consists of a non-homogeneous incompressible Navier-Stokes-Cahn-Hilliard system. In this talk, I will present some recent results concerning the propagation of regularity of global weak solutions and their longtime convergence towards an equilibrium state in three dimensional bounded domains.

**TBA**

*Zaher HANI, University of Michigan*

## **Homogenization methods applied to fluid-solid particle systems**

*Matthieu HILLAIRET, University of Montpellier*

In this talk I will consider ODE/PDE coupled systems describing the interactions between solid particles and a viscous fluid. When the particles are numerous and small it is more relevant to consider the cloud of particles as a continuum phase. It is then mandatory to derive interaction terms between fluid and solid phases. A formal method for this can be to start from a description of interactions between a single particle and a surrounding fluid and operate averaging operators. The aim of the talk is to present mathematical methods for justifying such computations with a special eye toward the computation of the effective velocity of a cloud of particles falling in a viscous fluid.

The talk will be based on recent results obtained in collaboration with R. Höfer (Univ. Regensburg).

## **Desingularization of time periodic vortices in bounded domains**

*Taoufik HMIDI, New York University Abu Dhabi*

In this talk, I will discuss some aspects of the vortex motion for Euler equations in the planar case. In the first part, I will review some results on the desingularization of time periodic point vortices in the rigid case. In the second part, we shall focus on vortex motion in bounded domains. Specifically, in a convex domain any single vortex performs a time periodic orbit. We will show that almost all the orbits can be desingularized into time-periodic concentrated patches. For this aim, we use KAM techniques combined with complex analysis tools.

This is a joint work with Zineb Hassainia and Emeric Roullely.

## **Regularity of vortex and SQG patches**

*Alexander KISELEV, Duke University*

Patch solutions are an important class of special singular solutions to the 2D Euler or surface quasi-geostrophic (SQG) equations that model evolution of regions of vorticity with sharp boundaries (like hurricanes) or sharp temperature fronts in atmosphere. I will discuss recent progress on regularity properties of vortex and SQG patches. In particular, I will present an example of a vortex patch with continuous initial curvature that immediately becomes infinite but returns to  $C^2$  class at all integer times only without being time periodic. The proof involves derivation of a new system describing the patch evolution in terms of arc-length and curvature. A similar approach leads to discovery of strong ill-posedness of the SQG patches in all but  $L^2$  based spaces, or spaces of infinitely smooth functions.

The talk is based on a work joint with Xiaoyutao Luo.



## **Infinitely many isolas of modulational instability for Stokes waves**

*Alberto MASPERO, SISSA Trieste*

We prove the long-standing conjecture regarding the existence of infinitely many high-frequency modulational instability “isolas” for a Stokes wave in arbitrary depth, subject to longitudinal perturbations. We completely describe the spectral bands with non-zero real part of the spectrum, away from the origin, of the water waves system linearized at a Stokes wave of small amplitude. The unstable spectrum is the union of isolas of elliptical shape, centered along the imaginary axis, and shrinking exponentially fast away from the origin.

This is a joint work with M. Berti, L. Corsi and P. Ventura.

## **On Euler equations with in-flow and out-flow boundary conditions**

*Anna Laura MAZZUCATO, Penn State University*

I will discuss recent results concerning the well-posedness and regularity for the incompressible Euler equations when in-flow and out-flow boundary conditions are imposed on parts of the boundary, motivated by applications to boundary layers.

This is joint work with Gung-Min Gie (U. Louisville, USA) and James Kelliher (UC Riverside, USA).

## **Non-uniqueness and energy dissipation for 2D Euler equations with vorticity in Hardy spaces**

*Stefano MODENA, Gran Sasso Science Institute*

We show by convex integration that uniqueness of solutions to the 2D incompressible Euler equations fails in the class of admissible (i.e. energy dissipating), compactly supported,  $L_t^\infty L_x^2$  velocity fields having vorticity in the real Hardy space  $H^p(\mathbb{R}^2)$ , for any  $p \in (0, 1)$ .

**TBA**

*Paolo SECCHI, University of Brescia*

# Junior Talks

## **A priori estimates for the 2D one-phase Muskat problem with contact angle**

*Edoardo BOCCHI, Polytechnic University of Milan*

We address the dynamics of a viscous and incompressible free-surface fluid that flows through a porous medium and interacts with a vertical wall. In the derivation of a priori estimates, the demand of higher-order regularity leads to the study of time-differentiated problems. It turns out that the structure of the nonlinear terms does not permit to close the estimates and we employ elliptic estimates for the velocity potential to improve the control provided by the energy and the dissipation and close the argument. Although the presence of a corner, we avoid weighted estimates by considering the Neumann-Neumann problem, which is less singular than the Dirichlet-Neumann problem.

This talk is based on a joint work in progress with Ángel Castro and Francisco Gancedo.

## **On the high compressible limit for the Navier-Stokes-Korteweg model with density dependent viscosity**

*Matteo CAGGIO, Czech Academy of Sciences*

The talk is devoted to the regime of high Mach number flows for compressible barotropic fluids with density dependent viscosity. The Korteweg model as an isothermal model of capillary fluids is considered. A weak-strong uniqueness analysis is also discussed.

## **Rotating fluids and forced fast oscillating waves**

*Luca FRANZOI, University of Milan*

The  $\beta$ -plane equation is an approximation model of the Euler-Coriolis equations for rotating fluid. The goal of this short talk is to give an overview on these equations and to quickly present a forthcoming result that studies the dynamics under the effect of a fast oscillating forcing term of large size, therefore not perturbative.

This is a joint work with Roberta Bianchini, Riccardo Montalto and Shulamit Terracina.

## **Wave kinetic theory for the forced/dissipated NLS equation**

*Ricardo GRANDE IZQUIERDO, SISSA Trieste*

We will present some recent developments in the justification of kinetic equations in the presence of forcing and dissipation. Such settings are of particular physical relevance as they allow the study of cascades: the transfer of energy from large scales to small scales. In this talk, we provide the first rigorous justification of such a kinetic equation in the case of a wave system governed by the cubic Schrödinger equation with a stochastic forcing and viscous dissipation. We will describe various regimes depending on the relative strength of the dissipation, the forcing and the nonlinear interactions, which give rise to different kinetic equations.

Based on joint work with Zaher Hani.

## Scaling laws in turbulent convection

*Camilla NOBILI, University of Surrey*

Scaling laws are a useful tool in studying and characterizing geophysical flows as they may indicate their behaviour in extreme parameter regimes which are unapproachable by experiments. In particular, the challenge of finding scaling laws requires synergistic efforts involving laboratory, computational, and theoretical studies. In fact, deducing and calibrating scaling laws requires physical arguments, analytical bounding arguments, numerical simulations and experiments. We will focus on convection problems as they are a relevant in a multitude of natural phenomena in meteorology, oceanography and industrial applications. In this seminar we are interested in rigorously proving power law scalings for the heat transport, as measured by the nondimensional Nusselt number  $Nu$  for the Rayleigh-Bénard convection problem. In this specific case the scaling laws have the functional form  $Nu \sim Ra^\alpha$ , where  $Ra$  is the thermal driving of the system. We will present the latest rigorous upper bounds on the Nusselt number for flows subject to various boundary conditions (no-slip, free-slip and Navier-slip) in flat and rough domains. In particular we will show how to substantially simplify arguments used in the seminal works of Doering and Constantin in the 90's and improve bounds.

## Global solutions to the Euler-Coriolis system

*Xiao REN, Peking University*

We prove the global well-posedness and scattering for the 3D incompressible Euler-Coriolis system with sufficiently small, regular and localized initial data. Equivalently, we obtain global asymptotic stability for the uniform rotating solutions to the pure Euler system. This extends a recent work of Guo, Pausader and Widmayer to the general non-axisymmetric setting. The proof involves a new inherent null type structure, a multiscale decomposition for bilinear forms and sharp dispersive estimates. Joint work with Gang Tian.